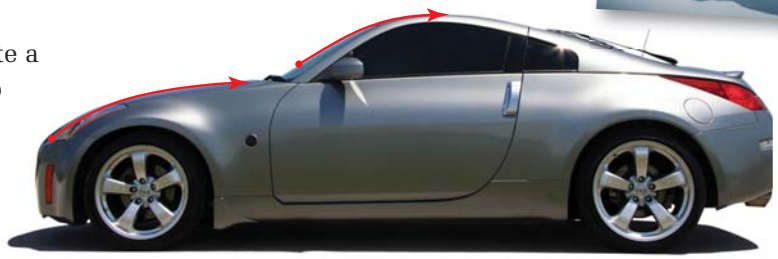
Unit 1 Project Wrap-Up

The Art of Mathematics

- Select a piece of artwork, a photo, or an image that clearly illustrates at least two different types of functions you have encountered in this unit, such as linear, absolute value, quadratic, radical, and polynomial.
- Determine function equations that model at least two aspects or portions of the image.
- Justify your choice of equations by superimposing them on the image.
- Display your piece of art. You may wish to use a poster, a PowerPoint presentation, a brochure, or some other format of your choice.

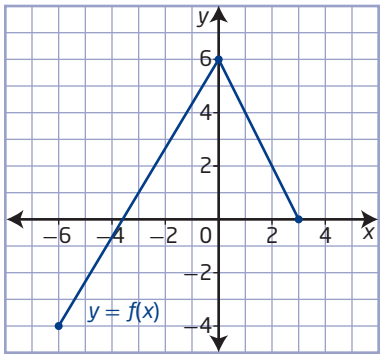
You may wish to create a class bulletin board to display your artwork.



Cumulative Review, Chapters 1–3

Chapter 1 Function Transformations

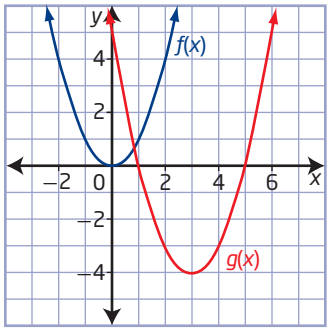
1. Given the graph of the function $y = f(x)$, sketch the graph of each transformation.



a) $y + 2 = f(x - 3)$ **b)** $y + 1 = -f(x)$

c) $y = f(3x + 6)$ **d)** $y = 3f(-x)$

2. Write the equation for the translated graph, $g(x)$, in the form $y - k = f(x - h)$.



3. Describe the combination of transformations that must be applied to the function $f(x)$ to obtain the transformed function $g(x)$.

a) $y = f(x)$ and $g(x) = f(x + 1) - 5$

b) $f(x) = x^2$ and $g(x) = -3(x - 2)^2$

c) $f(x) = |x|$ and $g(x) = |-x + 1| + 3$

4. The graph of $y = f(x)$ is transformed as indicated. State the coordinates of the image point of (6, 9) on the transformed graph.

a) $h(x) = f(x - 3) + 1$

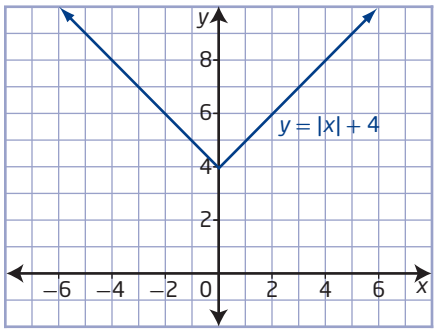
b) $i(x) = -2f(x)$

c) $j(x) = f(-3x)$

5. The x-intercepts of the graph of $y = f(x)$ are -4 and 6. The y-intercept is -3. Determine the new x-intercepts and y-intercept for each of the following transformations of $f(x)$.

a) $y = f(3x)$ **b)** $y = -2f(x)$

6. Consider the graph of $y = |x| + 4$.



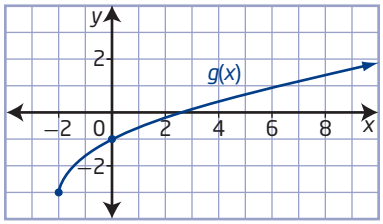
a) Does this graph represent a function?

b) Sketch the graph of the inverse of $y = |x| + 4$.

c) Is the inverse of $y = |x| + 4$ a function? If not, restrict the domain of $y = |x| + 4$ so that its inverse is a function.

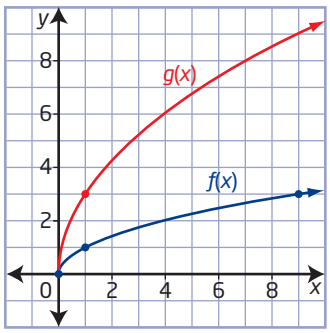
Chapter 2 Radical Functions

7. The graph of the function $f(x) = \sqrt{x}$ is transformed to the graph shown. Determine the equation of the transformed graph in the form $g(x) = \sqrt{b(x - h)} - k$.



8. The graph of the function $f(x) = \sqrt{x}$ is transformed by a vertical stretch by a factor of 2 and then reflected in the y-axis and translated 1 unit to the left. State the equation of the transformed function, sketch the graph, and identify the domain and range.

9. The graph of $g(x)$ is a transformation of the graph of $f(x)$.



a) Write the equation of $g(x)$ as a horizontal stretch of $f(x)$.

b) Write the equation of $g(x)$ as a vertical stretch of $f(x)$.

c) Show that the functions in parts a) and b) are equivalent.

10. Consider the functions $f(x) = x^2 - 1$ and $g(x) = \sqrt{f(x)}$.

a) Compare the x-intercepts of the graphs of the two functions. Explain your results.

b) Compare the domains of the functions. Explain your results.

11. The radical equation $2x = \sqrt{x + 3} - 5$ can be solved graphically or algebraically.

a) Ron solved the equation algebraically and obtained the solutions $x = -2.75$ and $x = -2$. Are these solutions correct? Explain.

b) Solve the equation graphically to confirm your answer to part a).

12. Consider the function $f(x) = 3\sqrt{x - 4} - 6$.

a) Sketch the graph of the function and determine its x-intercept.

b) Solve the equation $0 = 3\sqrt{x - 4} - 6$.

c) Describe the relationship between the x-intercept of the graph and the solution to the equation.

Chapter 3 Polynomial Functions

13. Divide each of the following as indicated. Express your answer in the form $\frac{P(x)}{x - a} = Q(x) + \frac{R}{x - a}$. Confirm your remainder using the remainder theorem.

a) $x^4 + 3x + 4$ divided by $x + 1$

b) $x^3 + 5x^2 + x - 9$ divided by $x + 3$

14. List the possible integral zeros of the polynomial $P(x) = x^4 - 3x^3 - 3x^2 + 11x - 6$. Use the remainder theorem to determine the remainder for each possible value.

15. Factor fully.

a) $x^3 - 21x + 20$

b) $x^3 + 3x^2 - 10x - 24$

c) $-x^4 + 8x^2 - 16$

16. Determine the x-intercepts and the y-intercept of the graphs of each polynomial function. Then, sketch the graph.

a) $f(x) = -x^3 + 2x^2 + 9x - 18$

b) $g(x) = x^4 - 2x^3 - 3x^2 + 4x + 4$

17. The volume of a box is represented by the function $V(x) = x^3 + 2x^2 - 11x - 12$.

a) If the height of the box can be represented by $x + 1$, determine the possible length and width by factoring the polynomial.

b) If the height of the box is 4.5 m, determine the dimensions of the box.

18. Determine the equation of the transformed function.

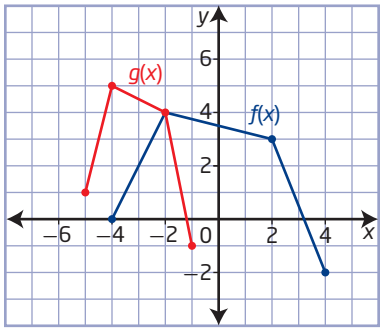
$f(x) = x^3$ is stretched vertically about the x-axis by a factor of 3, then reflected in the y-axis, and then translated horizontally 5 units to the right.

Unit 1 Test

Multiple Choice

For #1 to #7, choose the best answer.

1. The graph of $f(x)$ and its transformation, $g(x)$, are shown below.



The equation of the transformed function is

A $g(x) = f\left(\frac{1}{2}(x - 3)\right) + 1$

B $g(x) = f(2(x - 3)) + 1$

C $g(x) = f\left(\frac{1}{2}(x + 3)\right) + 1$

D $g(x) = f(2(x + 3)) + 1$

2. The graph of the function $y = f(x)$ is transformed by a reflection in the y-axis and a horizontal stretch about the y-axis by a factor of 3. Which of the following will not change?

I the domain

II the range

III the x-intercepts

IV the y-intercept

A I only

B I and III

C II and IV

D depends on $y = f(x)$

3. Which pair of functions are *not* inverses of each other?

A $f(x) = 5x$ and $g(x) = \frac{x}{5}$

B $f(x) = x + 3$ and $g(x) = x - 3$

C $f(x) = 4x - 1$ and $g(x) = \frac{1}{4}x + \frac{1}{4}$

D $f(x) = \frac{x}{2} + 5$ and $g(x) = 2x - 5$

4. Which function has a domain of $\{x \mid x \in \mathrm{R}\}$ and a range of $\{y \mid y \geq -3, y \in \mathrm{R}\}$?

A $y = |x + 4| - 3$

B $y = \sqrt{x + 4} - 3$

C $y = \sqrt{x^2 - 4} - 3$

D $y = (x - 4)^3 - 3$

5. If the graph of $y = \sqrt{x + 3}$ is reflected in the line $y = x$, then which statement is true?

A All invariant points lie on the y-axis.

B The new graph is not a function.

C The point (6, 3) will become (−3, 6).

D The domain of the new graph is $\{x \mid x \geq 0, x \in \mathrm{R}\}$.

6. If the graph of a polynomial function of degree 3 passes through (2, 4) and has x-intercepts of −2 and 3 only, the function could be

A $f(x) = x^3 + x^2 - 8x - 12$

B $f(x) = x^3 - x^2 - 8x + 12$

C $f(x) = x^3 - 4x^2 - 3x + 18$

D $f(x) = x^3 + 4x^2 - 3x - 18$

7. If $P(x) = -x^3 - 4x^2 + x + 4$, then

A $x + 1$ is a factor

B $P(0) = -1$

C the y-intercept is −4

D $x - 1$ is not a factor

Numerical Response

Copy and complete the statements in #8 to #11.

8. When $x^4 + k$ is divided by $x + 2$, the remainder is 3. The value of k is ■.

9. If the range of the function $y = f(x)$ is $\{y \mid y \geq 11, y \in \mathrm{R}\}$, then the range of the new function $g(x) = f(x + 2) - 3$ is ■.

10. The graph of the function $f(x) = |x|$ is transformed so that the point (x, y) becomes $(x - 2, y + 3)$. The equation of the transformed function is $g(x) =$ ■.

11. The root of the equation $x = \sqrt{2x - 1} + 2$ is ■.

Written Response

12. **a)** The graph of $y = x^2$ is stretched horizontally about the y-axis by a factor of $\frac{1}{2}$ and then translated horizontally 6 units to the right. Sketch the graph.

b) The graph of $y = x^2$ is translated horizontally 6 units to the right and then stretched horizontally about the y-axis by a factor of $\frac{1}{2}$. Sketch the graph.

c) How are the two images related? Explain.

13. Consider $f(x) = x^2 - 9$.

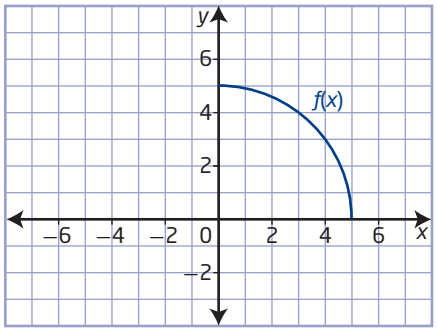
a) Sketch the graph of $f(x)$.

b) Determine the equation of the inverse of $f(x)$ and sketch its graph.

c) State the equation of $y = \sqrt{f(x)}$ and sketch its graph.

d) Identify and compare the domain and range of the three relations.

14. The graph of $y = f(x)$ represents one quarter of a circle. Describe the reflections of $y = f(x)$ required to produce a whole circle. State the equations required.



15. Mary and John were asked to solve the equation $2x = \sqrt{x + 1} + 4$.

a) Mary chose to solve the equation algebraically. Her first steps are shown. Identify any errors in her work, and complete the correct solution.

$$2x = \sqrt{x + 1} + 4$$

Step 1: $(2x)^2 = (\sqrt{x + 1} + 4)^2$

Step 2: $4x^2 = x + 1 + 16$

b) John decided to find the solution graphically. He entered the following equations in his calculator. Could his method lead to a correct answer? Explain.

$y = \sqrt{x + 1} + 4$

$y = 2x$

16. Given that $x + 3$ is a factor of the polynomial $P(x) = x^4 + 3x^3 + cx^2 - 7x + 6$, determine the value of c. Then, factor the polynomial fully.

17. Consider $P(x) = x^3 - 7x - 6$.

a) List the possible integral zeros of $P(x)$.

b) Factor $P(x)$ fully.

c) State the x-intercepts and y-intercept of the graph of the function $P(x)$.

d) Determine the intervals where $\mathrm{P}(x) \geq 0$.